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By

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Calculation of Factors, Affecting the Accuracy in Determining the Drift of Floating Integrating Gyroscopes

by

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We will assume, that when testing for drift the investigated gyroscope and platform of the stand are situated, as is shown in fig.1.For the sake of commonness we will assume that the platform of the stand is driven (brought into rotation) through a gap-less reduction gear. The motor of the platform is coupled to an amplifier to the input of which is fed the output voltage of the sensing element of the angle of the investigated gyroscope, fastened to the platform of the stand (fig.1).

In fig.1. and further on were adopted the following three systems of axis, originating in point 0 where the axes of rotation of frame and rotor of examined gyroscope do intersect:

1.The system of xyz_0 axes is connected with the body of the investigated gyroscope. The x and y axes are the output and input axes of the gyroscope;axis z_0 is perpendicular to axes x, y and forms together with them a right trihedral.

SEE PAGE 1a FOR FIGURE 1.

Fig.1.Variant of platform and gyroscope arrangement when checking drift of gyroscope.

2. The system of xi eta zeta axes, connected with the earth and is oriented geographically. Axes chi and eta are horizontal; axis chi faces eastward, axis eta - northward, and axis zeta into the zenith.

3. The system of XYZ axes is connected with the platform of the testing stand. Axis Y appears to be the axis of rotation of the platform; axes X and Z are perpendicular to axis Y and form with it a right trihedral .

In fig.1 and further on β designates the angle of inclination of the figurational

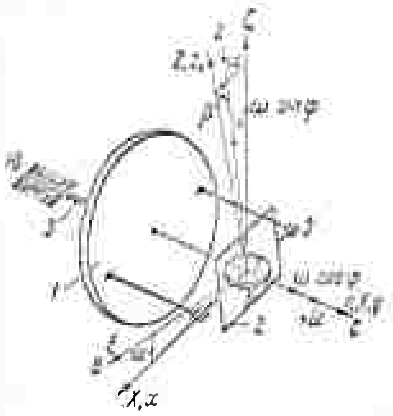

Figure 1.

axis of the gyroscope z from axis z_0; a and α - the angle and angular velocity of platform rotation about axis Y. Omega designates the angular velocity of diurnal rotation of the Earth, and φ - latitude of test stand disposition.

In an ideal case when testing according to fig.1. the axes y and Y should coincide with axis eta; axes X and Z should lie in plane xi zeta. Axis x should coincide with axis X, and z_0 with axis Z.

The moment of interferences M, affecting the gyroscope around axis x, will be assumed as consisting of constant and variable components. The constant component will be designated by M^0. The variable component, which is to a large extent and accidental value, to simplify the analysis will be considered as varying in accordance with the harmonics law with amplitude εM^0, frequency p and initial phase θ. The coefficient ε is equal to the ratio of amplitude of variable component of moment M to it constant component M^0. In this way, considering moment M as positive, when the direction of its vector coincides with positive direction of axis x, it is assumed that the moment of interferences M is a function of time t and is determined by formula

$$M = M^0\left[1 + \varepsilon \sin (pt + \theta)\right]$$

In this case the angular velocity of the drift ω_1 will also be a function of time, whereby its instantaneous values will be determined by equation

$$\omega_1 = \omega_1^0[1 + \varepsilon \sin(pt + \theta)] \quad \left(\omega_1^0 = \frac{M^0}{H}\right) \tag{1}$$

Here ω_1 - constant component of angular velocity of drift (H - eigen-moment of gyroscope).

Assuming that the tested gyroscope, amplifier and platform motor are inertialess we can write their equation in the following form :

Equation of gyroscope (when compiling same we have disregarded the gyroscopic moment, due to the projection of velocity ω on axis z_0, because this moment is small in comparison with other moments because the ordinary angle $\beta \approx 0$)

$$U_-^{(1)} = K\{\omega_y + \dot{\alpha} - \omega_1^0[1 + \varepsilon \sin(pt + \theta)]\}, \quad K = \frac{HK_1}{K_2} \tag{2}$$

Equation of amplifier

$$U_-^{(2)} = K_3[1 + \delta_1 \sin(\nu_1 t + \theta_1)]\, U_-^{(1)} \tag{3}$$

Equation of motor

$$\dot{\alpha}_3 = -K_4 U_-^{(2)} \tag{4}$$

Here K_2 - specific damping moment of gyroscope ; K_1 and $U_-^{(1)}$ - curvature of characteristic (sensitivity) and output voltage of gyroscope sensing element; ω_y - projection of angular velocity of diurnal rotation of the Earth ω on axis y; K_3 and $U_-^{(2)}$ - nominal (rated) coefficient of amplification and output voltage of amplifier; K_4- curvature of velocity characteristic of motor; α_3 - angle of rotation of motor shaft.

Equation (3) takes into consideration the possibility of fluctuations of the amplification factor relative to its nominal value with amplitude δK_3 (δ_1= const - small parameter) and frequency ν_1; θ_1 designates the initial phase.

Because of error in reduction gear the platform during uniform rotation of motor shaft will rotate irregularly. The dependence of the angle of rotation of stand platform upon the angle of rotation of motor shaft is determined by expression

$$\alpha = \frac{\alpha_3}{i} + \delta_2 \sin(\nu_2 \alpha + \theta_2) \tag{5}$$

Here i - nominal value of reductor gear ratio; δ_2 = const - small parameter, equal to the maximum absolute error in angle of rotation of the platform, due to errors of the reduction gear; ν_2 - dimensionless frequency ; θ_2 - initial phase.

Having solved equation (3) relative to $U_-^{(1)}$ and substituting in it subsequently equations (4) and (5), we further assume in it

$$1 / [1 + \delta_1 \sin(\nu_1 t + \theta_1)] \approx 1 - \delta_1 \sin(\nu_1 t + \theta_1) \quad 5a$$

and disregard the member, containing products of small values $\delta_1 \delta_2$. We differentiate the expression for $U_-^{(1)}$ obtained in this way once with respect of time and substitute in equation (2)

In consequence the equation of motion of the platform (stand platform) will be obtained in following form:

$$T\ddot{\alpha} + \dot{\alpha} = -\omega_y + \omega_1^{\circ} + \varepsilon\omega_1^{\circ} \sin(pt + \theta) +$$
$$+ \delta_1 T \frac{d}{dt}[\dot{\alpha} \sin(\nu_1 t + \theta_1)] + \delta_2 T \nu_2 \frac{d}{dt}[\dot{\alpha} \cos(\nu_2 \alpha + \theta_2)] \tag{6}$$

where the time constant

$$T = \frac{i}{KK_3K_4} \tag{7}$$

Integrating equation (6) for the first time at initial conditions $\alpha = \dot{\alpha} = 0$ and at t = 0, we will obtain

$$T\dot{\alpha} + \alpha = -(\omega_y - \omega_1{}^\circ)t - \frac{\varepsilon\omega_1{}^\circ}{p}[\cos(pt + 0) - \cos 0] +$$
$$+ T\dot{\alpha}[\delta_1 \sin(\nu_1 t + 0_1) + \delta_2\nu_2 \cos(\nu_2\alpha + 0_2)] \tag{8}$$

The given nonlinear equation will be integrated approximately (but with a fully sufficient degree of accuracy), assuming that in its right side

$$\alpha = -(\omega_y - \omega_1{}^\circ)t \text{ и } \dot{\alpha} = -(\omega_y - \omega_1{}^\circ) \quad 8a$$

One can easily see that these values for alpha and alpha are derived in stable state, provided $\varepsilon = \delta_1 = \delta_2 = 0$; when $T = \varepsilon = 0$ this would happen everywheres. Having completed the integration, we obtain :

$$\alpha = -(\omega_y - \omega_1{}^\circ)(t - T) - \frac{\varepsilon\omega_1{}^\circ}{p}[\sin 0^* \sin(pt + 0 + 0^*) - \cos 0] +$$
$$+ T(\omega_y - \omega_1{}^\circ)\{\delta_1 \sin 0_1{}^* \cos(\nu_1 t + 0_1 + 0_1{}^*) -$$
$$- \delta_2\nu_2 \sin 0_2{}^* \sin[\nu_2(\omega_y - \omega_1{}^\circ)t - 0_2 + 0_1{}^*]\} -$$
$$- \left\{\frac{\varepsilon\omega_1{}^\circ}{p}\cos 0^* \cos(0 + 0^*) + T(\omega_y - \omega_1{}^\circ) \times\right.$$
$$\left.\times [1 + \delta_1 \sin 0_1{}^* \cos(0_1 + 0_1{}^*) + \delta_2\nu_2 \sin 0_2{}^* \sin(0_2 - 0_2{}^*)]\right\} e^{-(t/T)} \tag{9}$$

Here

$$\operatorname{ctg} 0^* = Tp, \qquad \operatorname{ctg} 0_1{}^* = T\nu_1, \qquad \operatorname{ctg} 0_1{}^* = T\nu_2(\omega_y - \omega_1{}^\circ) \tag{10}$$

From expression (9) is evodent that the time constant T should be possibly smaller. When $\varepsilon = \delta_1 = \delta_2 = 0$ in stable state

$$\dot{\alpha} = -(\omega_y - \omega_1{}^\circ) \quad 9a$$

In this case from equations (5), (4), (3), considering that $U_-^{(1)} = K_1\beta$ and using equation (7), we obtain

$$T = \frac{\beta}{N(\omega_y - \omega_1{}^\circ)} \qquad \left(N = \frac{H}{K_2}\right) \tag{11}$$

It is evident therefrom, that T will be smaller the smaller the angle β will be, the engle necessary to produce the rotating motion of the platform of the stand to revolve at an angular velocity $-(\omega_y - \omega_1{}^\circ)$. In addition, T is inversely proportional to coefficient N, which appears to be the parameter of the gyroscope.

We will assume, that in the moments of time t_1 and t_2 ($t_2 > t_1$, $t_2 - t_1 = \tau$) the angle α was equal to α_1 and α_2 respectively. Then using expression (9), assuming that $t_1 \gg T$, and disregarding on the basis of this the member, containing exp $(-t_2/T)$ -exp $(-t_1/T)$, we obtain :

$$-\omega_1^* + \omega_y + \frac{\alpha_2 - \alpha_1}{\tau} + \Delta_1 + \Delta_2 + \Delta_3 = 0 \qquad (12)$$

In equation (12) have been adopted the following designations

$$\omega_1^* = \omega_1^\circ\left\{1 - \frac{\varepsilon}{p\tau}\left[\cos(pt_2 + \theta) - \cos(pt_1 + \theta)\right]\right\} \qquad (13)$$

$$\Delta_1 = \frac{\varepsilon\omega_1^\circ}{p\tau}\cos\theta^*\left[\sin(pt_2 + \theta - \theta^*) - \sin(pt_1 + \theta - \theta^*)\right] \qquad (14)$$

$$\Delta_2 = -\frac{T\delta_1}{\tau}(\omega_y - \omega_1^\circ)\sin\theta_1^*\left[\cos(\nu_1 t_2 + \theta_1 + \theta_1^*) - \cos(\nu_1 t_1 + \theta_1 + \theta_1^*)\right] \qquad (15)$$

$$\Delta_3 = \frac{T\delta_2\nu_2}{\tau}(\omega_y - \omega_1^\circ)\sin\theta_2^*\{\sin[\nu_2(\omega_y - \omega_1^\circ)t_2 - \theta_2 + \theta_2^*] - $$
$$- \sin[\nu_2(\omega_y - \omega_1^\circ)t_1 - \theta_2 + \theta_2^*]\} \qquad (16)$$

Through ω_1^* was designated the actual mean integral value of the angular velocity of the drift during time τ. From equation (13) is evident that this velocity depends upon τ and upon t_1 as well, which appears to be a random value. The maximum ω_1^* value is obtained at $t_1 = (2\pi n - \theta)/p$ ($n = 0, 1, 2, \ldots$) and $\tau = (2k+1)\pi/$ ($k=0,1,2,\ldots$); in this case it will be equal to

$$\omega_1^*{}_{max} = \omega_1^\circ\left(1 + \frac{2\varepsilon}{p\tau}\right) = \omega_1^\circ\left[1 + \frac{2\varepsilon}{(2k+1)\pi}\right] \quad 16a$$

The minimum value ω_1^*, obtainable at the very same values τ, but at $t_1 = [(2n+1)\pi - \theta]/p$, equals

$$\omega_1^*{}_{min} = \omega_1^\circ\left(1 - \frac{2\varepsilon}{p\tau}\right) = \omega_1^\circ\left[1 - \frac{2\varepsilon}{(2k+1)\pi}\right] \quad 16b$$

In this way we have $\omega_1^*{}_{min} \leq \omega_1^* \leq \omega_1^*{}_{max}$. Whereby

$$m = \omega_1^*{}_{max} - \omega_1^*{}_{min} = \frac{4\varepsilon\omega_1^\circ}{p\tau} = \frac{4\varepsilon\omega_1^\circ}{(2k+1)\pi} \qquad (17)$$

With rise in τ m decreases, and ω_1^* tends toward ω_1^*.

If the stand would be ideal (for this, under the assumptions made by us, it is necessary to have $T = \delta_1 = \delta_2 = 0$, then $\Delta_1 = \Delta_2 = \Delta_3 = 0$), and ω_y, α_1, α_2 and τ we known to be absolutely accurate, then the velocity values ω_1^*, calculated by formula

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$$\omega_1^* = \omega_y + \frac{\alpha_2 - \alpha_1}{\tau} \qquad (18)$$

obtainable from expression (12), would be equal to its actual values. The angle $alpha_1$ and $alpha_2$ should be substitutedin formula (18) with consideration of their sign. The sign of the angle alpha should be identical with the velocity sign alpha, which is considered to be positive, when the direction of its vector coincides with the positive direction of axis Y (fig.1)During tests $\omega_y \geq 0$, if $\omega_y > 0$, then angle $a < 0$ (see term (9)).

It is clear from the preceding, that -- even during the fulfillment of the men tioned ideal conditions of value ω_1^*, obtainable during repeated measurements, realized within one and the same time τ, they will be different as result of the dependence of ω_1^* upon t_1. As is evident fromequation (17) the magnitude of total disagreement of values ω_1^* will not exceed

$$m_{max} = \frac{4\varepsilon\omega_1^\circ}{\pi} \qquad 18a$$

In actuality T, $\delta_1$and δ_2 are not equal to zero and the adopted value ω_y and the obtainable by measurement values $alpha_1$ and $alpha_2$ and τ differ from the actual values by certain small values $\Delta\omega_y$, $\Delta\alpha_1$, $\Delta\alpha_2$ and $\Delta\tau$. We will explain, how this reflects itself on the accuracy of determining ω_1 by formula (18). We will analyze the right side of equation (18) into a Taylor series po powers $\Delta\omega_y$, $\Delta\alpha_1$ $\Delta\alpha_2$ and $\Delta\tau$,confining ourselves to members, containing the given small values in zero and first degree. Utilizing the obtained expansion and formula (12), we obtain, that the absolute error in the determination of ω_1^* according to formula (18) is equal

$$\Delta\omega_1' = \Delta_1 + \ldots + \Delta_7 \qquad 18b$$

where

$$\Delta_4 = \Delta\omega_y, \qquad \Delta_5 = -\frac{\Delta\alpha_1}{\tau}, \qquad \Delta_6 = \frac{\Delta\alpha_2}{\tau}, \quad \Delta_7 = -\frac{\alpha_2 - \alpha_1}{\tau^2}\Delta\tau \approx \frac{\omega_y - \omega_1}{\tau}\Delta\tau \qquad 18c$$

and Δ_1, Δ_2, Δ_3 are determined by formulae (14), (15), (16). Their maximum values are equal to :

$$\Delta_{1l} = \pm 2\varepsilon\omega_1^\circ \frac{T}{\tau}, \quad \Delta_{2l} = \pm 2\delta_1(\omega_y - \omega_1^\circ)\frac{T}{\tau}, \quad \Delta_{3l} = \pm 2\delta_2\nu_2(\omega_y - \omega_1^\circ)\frac{T}{\tau} \qquad 18d$$

The maximum absolute error in determining ω_1^* by formula (18)can be assumed to

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$$\Delta\omega_{1l}^{*} = \pm(\Delta_{1l} + \ldots + \Delta_{7l})^{\frac{1}{2}} \quad 18a$$

Having substituted here the values $\Delta_{1l}, \ldots, \Delta_{7l}$, we obtain:

$$\Delta\omega_{1l}^{*} = \pm\frac{1}{\tau}\Big\{\Delta\omega_{yl}^{2}\tau^{2} + 2\Delta\alpha_{l}^{2}\,\omega + (\omega_{y} - \omega_{1}^{\circ})^{2} \times$$

$$\times\Big[\Delta\tau_{l}^{2} + 4T^{2}\Big(\delta_{1}^{2} + \delta_{2}^{2}\nu_{2}^{2} + \frac{\varepsilon^{2}\omega_{1}^{\circ 2}}{(\omega_{y} - \omega_{1}^{\circ})^{2}}\Big)\Big]\Big\}^{\frac{1}{2}} \qquad (19)$$

where $\Delta\omega_{yl}$ and $\Delta\gamma l$ - maximum values of absolute errors $\Delta\omega_{y}$ and $\Delta\gamma$; maximum values of absolute errors $\Delta\alpha_{1}$ and $\Delta\alpha_{2}$ assumed to be identical, equalling $\Delta\alpha_{l}$.

If it is necessary, that the maximum absolute error in determining ω_{1}^{*} by formula (18) should not exceed a certain given value, for which we will preserve designation $\Delta\omega_{1l}^{*}$, then time γ should satisfy the following condition, obtainable from formula (19)

$$\tau \geqslant \left(\frac{2\Delta\alpha_{l}^{2} + (\omega_{y} - \omega_{1}^{\circ})^{2}\{\Delta\tau_{l}^{2} + 4T\,[\delta_{1}^{2} + \delta_{2}^{2}\nu_{2}^{2} + \varepsilon^{2}\omega_{1}^{\circ 2}/(\omega_{y} - \omega_{1}^{\circ})^{2}]\}}{\Delta\omega_{1l}^{*2} - \Delta\omega_{yl}^{2}}\right)^{\frac{1}{2}} \qquad (20)$$

Hence it is evident, that the inequality should be fulfilled

$$\Delta\omega_{yl} < \Delta\omega_{1l}^{*} \qquad (21)$$

When this inequality is not satisfied no greater time γ will be able to assure determination of drift with required accuracy, characterized by value $\Delta\omega_{1l}^{*}$. In this case the actual value $\Delta\omega_{1l}^{*}$ will always be greater than $\Delta\omega_{y}l$, and at greater values γ it will be practically equal to $\Delta\omega_{yl}$, qhich plainly evident from formula (19).

When inequality (21) is satisfied there will everywheres exist such a time value γ, at which the maximum absolute error in determining ω_{1}^{*} by formula (18) will not exceed the given value $\Delta\omega_{1l}^{*}$. In this case the required time value τ will be the lower the higher the satisfaction of inequality (21). Consequently the value $\Delta\omega_{y}l$ should be reduced by all possible means.

We will determine the value $\Delta\omega_{yl}$ for the case of testing drift, according to fig.1. The position of the platform, and consequently, the axes X, Y, Z connected with it relative to axes xi, eta, zeta, will be determined with the aid of angles ψ, ϑ, α (fig.2). The angle ψ is the angle formed by the projection of axis Y on the plane of horizon with plane of the meridian; ϑ- angle of inclination of axis Y from plane of the horizon; α- angle of rotation of the platform around axis Y.

When $\psi = \vartheta = \alpha = 0$ the axses X,Y,Z coincide with axes xi, eta, zeta respectively .

We will assume, that the tested instrument is fastened on the platform in corresponding fixing elements and that in general case the position of its axes x, y, z_0 relative to axes X,Y,Z is determined by three angles γ, λ, χ (fig.2). Gamma - angle formed by the projection of axis y with plane XY, χ- angle of rotation of instrument body around axis y. In the process of testing the angles ψ, ϑ, γ, λ, χ remain unchanged.

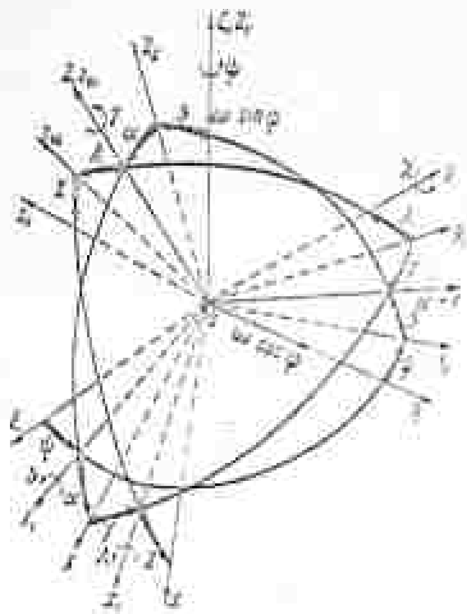


Fig.2.To determine magnitude of maximum absolute error $\Delta\omega_y \ell$

It is evident when examining fig 2, that the projection of angular velocity of the diurnal rotation of the Earth ω on the input axis y of the instrument

$$\omega_y = \omega \cos\varphi \cos(\eta, y) + \omega \sin\varphi \cos(\zeta, y) \tag{22}$$

where

$$\cos(\eta, y) = -[(\sin\psi\cos\alpha + \cos\psi\sin\vartheta\sin\alpha)\sin\gamma - \cos\psi\cos\vartheta\cos\gamma]\cos\lambda + \\ + (\sin\psi\sin\alpha - \cos\psi\cos\vartheta\cos\alpha)\sin\lambda \tag{23}$$

$$\cos(\zeta, y) = (\cos\vartheta\sin\alpha\sin\gamma + \sin\vartheta\cos\gamma)\cos\lambda + \\ + \cos\vartheta\cos\alpha\sin\lambda \tag{24}$$

From expressions (22) - (24) is evident that ω_y does not depend upon —— angle χ . But this still does not mean ,that no attention should be paid to the magnitude of and χ. It must be remembered , that upon a change of this angle there is also a change in position of the instrument relative to the field of gravitational force, and consequently, there will also be a change in its drift. It is évident from these expressions, that $\omega_y = \omega \cos\varphi$ at $\psi = \vartheta = \gamma = \lambda = 0$ and any given value α. This value ω_y, equal to the horizontal component of the diurnal rotation of the Earth, appears to be for the case under question the nominal value ω_y , substituted in formula (18) during the calculation of angular velocity of drift ω_l^*. Upon changing angle alpha as well as angle χ, there is a change in position of the instrument relative to the field of gravitation, and consequently, there is also a change in its drift.

Ordinarily angles ψ, ϑ, γ and λ appear to be so small, that if they are expressed in radians, they can at least be considered as small values of first order relative to unity. As to the angle alpha, it can generally not be considered as small.

Decomposing the right side of equation (22) into a Taylor series by degrees of small values $\psi, \vartheta, \gamma, \lambda$ and $\Delta\varphi$ ($\Delta\varphi$ - absolute error of determining latitude of point φ) in the zone $\psi = \vartheta = \gamma = \lambda = 0$, $\varphi = \varphi$ and confining ourselves to members, containing $\psi, \vartheta, \gamma, \lambda, \Delta\varphi$ only in zero and first degrees, we obtain

$$\omega_y = \omega \cos\varphi + \omega \sin\varphi\,(\vartheta + \gamma \sin\alpha + \lambda \cos\alpha - \Delta\varphi) \quad 24a$$

It is eviden herefrom that in case under consideration the absolute error in determining ω_y equals

$$\Delta\omega_y = \omega_y - \omega\cos\varphi - \omega\sin\varphi\,(\vartheta + \gamma\sin\alpha + \lambda\cos\alpha - \Delta\varphi) \quad 24b$$

The maximum value of this error can be accepted as equal

$$\Delta\omega_{yl} = \pm\,\omega\sin\varphi\sqrt{\vartheta_l^2 + \gamma_l^2\sin^2\alpha + \lambda_l^2\cos^2\alpha + \Delta\varphi_l^2} \quad 24c$$

where $\vartheta_l, \gamma_l, \lambda_l$ and $\Delta\varphi_l$ - maximum values of $\vartheta, \gamma, \lambda$ and $\Delta\varphi$.

The values $\Delta\omega_{yl}$ for the most widely known variants of testing floating integrating gyroscopes for drift, calculated by an analogous method, are listed in table given below. At the fifth and sixth testing variants the angle alpha remains small.

At the beginning of the report we have assumed, that the reduction gear of the test stand appears to be gapless. It is apparent that this is absolutely necessary for the --- fifth and sixth test variants, because in these cases the instantaneous angular velocity of the stand platform is equal to the instantaneous angular velocity of the drift of the examined gyroscope, which can generally be changed not only in value, but also in sign. At remaining five test variants the platform of the testing stand rotates in one direction with instantaneous angular velocity

$$\dot{\alpha} = -\omega_y + \omega_1 \quad 24d$$

which can change only somewhat in value on account of change in magnitude, and in general case also in sign of instantaneous angular velocity of drift ω_1.

It is evident from the table, that at the first four test variants the maximum absolute error $\Delta\omega_y l$, calculated with an accuracy of up to small values of first magnitude, is determined only by ϑ_l-values, $\Delta\varphi l$, γl, λl values and appears to be a function of φ and α angles. In case of the fifth and sixth variants the value $\Delta\omega_{yl}$ is affected also by ψ_l, but from angle α and $\Delta\varphi l$ the maximum absolute error $\Delta\omega_{yl}$ is independent in these cases. At the seventh variant $\Delta\omega_{yl} = 0$ with an accuracy to small values of first magnitude.

We will assume, that $\Delta\varphi l = \psi_l = \vartheta_l = \lambda l = \gamma l = q$; then : for the first two variants

$$\Delta\omega_{yl} = (\Delta\omega_{yl})_{1,2} \equiv \pm q\sqrt{3}\,\omega\sin\varphi \tag{25}$$

for third and fourth variants

$$\Delta\omega_{yl} = (\Delta\omega_{yl})_{3,4} \equiv \pm q\sqrt{3}\,\omega\cos\varphi \tag{26}$$

for fifth and sixth variants

$$\Delta\omega_{yl} = (\Delta\omega_{yl})_{5,6} \equiv \pm q\sqrt{2}\,\omega \tag{27}$$

Superimposing equation (25) over (27), we obtain

$$\frac{(\Delta\omega_{yl})_{1,2}}{(\Delta\omega_{yl})_{5,6}} = \sqrt{\frac{3}{2}}\sin\varphi = 1.23\sin\varphi \tag{28}$$

At a latitude $\varphi^* = 54°20'$ we have

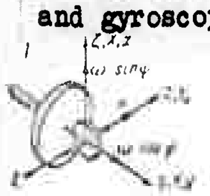
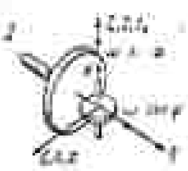
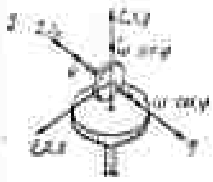
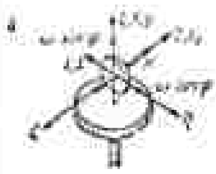
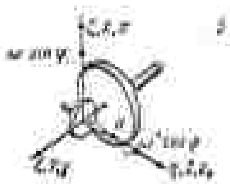
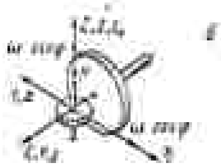
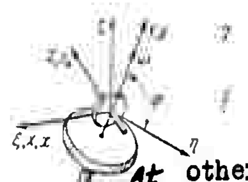
$$(\Delta\omega_{yl})_{1,2} = (\Delta\omega_{yl})_{5,6} \tag{29}$$

On all --- latitudes $\varphi > \varphi^*$ $(\Delta\omega_{yl})_{5.6} < (\Delta\omega_{yl})_{1.2}$ and, consequently, the fifth and sixth test variants are more preferred, than the first and second variants.

A characteristic feature of the seventh variant appears to be this that in this case the practically maximum absolute error $\Delta\omega_{yl} = 0$.

The analysis made gives basis of arriving at a conclusion in selecting the time interval τ between two subsequent measurements of angular position of the platform of the test stand when determining the drift of floating integrating gyroscopes. This time should be selected by formula (20). The value $\Delta\omega_{yl}$ should be determined by the corresponding formula shown in the rable or by one of the corresponding formulas (25)-(27).

Table. Nominal values ω_y and values of maximum absolute error $\Delta\omega_{yl}$ for various variants of platform and gyroscope arrangements during testing of floating integrating gyroscopes for drift.

Nominal scheme of platform and gyroscope arrangement	Nominal value ω_y	Value of absolute maximum error : $\Delta\omega_{yl}$
	$\omega\cos\varphi$	$\pm\omega\sin\varphi(\Delta\varphi_l^2+\vartheta_l^2+\gamma_l^2\cos^2\alpha+\lambda_l^2\sin^2\alpha)^{1/2}$
		$\pm\omega\sin\varphi(\Delta\varphi_l^2+\vartheta_l^2+\gamma_l^2\sin^2\alpha+\lambda_l^2\cos^2\alpha)^{1/2}$
	$\omega\sin\varphi$	$\pm\omega\cos\varphi(\Delta\varphi_l^2+\vartheta_l^2+\gamma_l^2\sin^2\alpha+\lambda_l^2\cos^2\alpha)^{1/2}$
		
	ω	$\pm\omega\sin\varphi[\vartheta_l^2+\lambda_l^2+(\psi_l^2+\gamma_l^2)\operatorname{ctg}^2\varphi]^{1/2}$
		$\pm\omega\sin\varphi[\vartheta_l^2+\gamma_l^2+(\psi_l^2+\lambda_l^2)\operatorname{ctg}^2\varphi]^{1/2}$
	ω	0

ϑ_l *MAXIMUM VALUE of angle of inclination of axis Y to plane of horizon.

ϑ_l - maximum value of angle of inclination of axis Y to vertical line

ϑ_l and ψ_l -maximum values of angles of inclination of axis Y to the plane of the horizon and to plane xi zeta.

At other conditions being equal for the seventh time variant γ will be the lowest, because in this case $\Delta\omega_{y1} = 0$. At the fifth and sixth variants $\omega_y = 0$ and consequently for their time γ at other conditionsbeing equal, will be somewhat smaller than for the first four variants.

Fundamentally the time magnitude γ is determined by values $\Delta\alpha_{\ell}$, $\Delta\omega_{1\ell}$ and $\Delta\omega_{y}\ell$. Consequently, $\Delta\alpha_{\ell}$ and $\Delta\omega_{y}\ell$ should be reduced by all means possible. It is very im portant insetad of separate measuring two angles α_1 and α_2 to measure only one angle, equalling to the difference $\alpha_2 - \alpha_1$, because this allows to reduce the time γ by approximately $\sqrt{2}$ times.

If the time γ corresponds to formula (20), then the experimental value of the mean integral angular velocity of the drift during the time γ equals

$$\omega_1^{**} = \omega^*_{1(18)} \pm \Delta\omega^*_{1\ell} \quad (30)$$

where $\omega^*_1(18)$ - value of the velocity $\overset{*}{\omega}1$, determined by formula (18) and $\Delta\omega_{1\ell}$ - value of maximum absolute error in determining $\overset{*}{\omega}_1$ by formula (18), given by equation (19).

In conclusion we wish to mention, -------that the author knows of no investigations, devoted directly to problems discussed in this report. Reports which do have some kind of close relationship to the subject in discussion is the work, e.g. of [1] .

Submitted Dec.26,1960

Literature

1. Draper, C. S., Wrigley, W., Grohe, L. R. - The Floating Gyro and Its Application to Geometrical, Stabilization Problems on Moving Bases. Aeronaut. Engng. Rev., 1956, V. 15, No. 6.

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